Playing the game of Go in an integrated Mathematics and Computing course*

Attila Egri-Nagy
Akita International University

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Abstract

Acquiring knowledge in Mathematics and Computing is increasingly needed in almost all areas of study. At the same time, it is difficult to motivate these subjects for students in other majors. One reason could be the lack of naturally arising interest; questions, for which the students genuinely seek answers. Playing the ancient game of Go can be the source of motivating problems, and the game itself can provide a shared base experience for the whole class.

Here we describe the incentives for, and the design decisions in developing an integrated artificial intelligence course centered around the game. By its logic, this potent combination leads to self-reflection and metacognition techniques. Transferring these skills could also help students in other subjects.

Due to rapid technological (the advance of automation) and societal changes (demographic shifts, declining university enrollments), education on all levels, admittedly or not, is in an existential crisis. How to prepare students for their future life? What to teach and how? No one can predict the future job market. Still, there are some short-term strategies

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(teaching what is needed at the moment, e.g. mathematical and computing skills) and long-term ideas for nurturing abilities for coping with constant change (Harari, 2018). We will address both issues. First, we try to identify a factor contributing to current failures in mathematical (epitomized in Lockhart and Devlin, 2009) and computational subjects.

Go players are keen on reasoning for the benefits of playing the game, since we tend to share what we enjoy. This paper can also be viewed as such an argument, with a particular focus on education.

1 Motivating studies

It is an everyday observation that learning could feel effortless if someone has a genuine interest in a subject. We take this as our central assumption for improving the teaching and learning process in an undergraduate mathematics and computing course. This is, of course, a simplified way of looking at the problem of learning, since there are several conditions other than motivation for achieving an optimal experience, a flow state (Csikszentmihalyi, 2009).

1.1 External motivation does not transfer to internal

Students may be very well motivated in their studies, e.g. preparing for an entrance exam or working towards a degree. However, these external incentives may not automatically become everyday interests in particular subjects. Courses in mathematics and computing are particularly prone to this type of failure. External pressures are high for passing standardized tests. Math anxiety develops very early (Sokolowski & Ansari, 2017). High-paid software engineering jobs are luring, but they require expertise in programming. Skill for writing computer code is known to be difficult to obtain (Jenkins, 2002), and it is usually hard-earned by countless hours of work. Without enjoying the coding assignments, it could become a painful activity. Computing subjects in general can be difficult to study for students without a genuine interest (either innate or developed) in symbolic languages and in computers' inner workings. Consequently, the learning process could lose much of its efficiency in terms of time versus the mastery of a skill.

Similar problems arise from the educator's perspective. It is not efficient to teach someone a method of solving a problem, who does not happen to have that particular problem. It is also not exactly a nice thing to do, since it often involves exercising power to force the person to pay attention. If everything else fails in a lecture, the instructor can still say that the exam will have questions of the kind being discussed to convince students to work on the problems. Traditional mathematics education works mostly this way (Lockhart & Devlin, 2009). The assumption is, that the algorithms we teach will be useful for the students at some later stage of their studies or subsequent professional work. This reasoning, no matter how correct it is, does not stop questions like 'Where am I going to use this?'. A traditional math class is a bit like selling a useless product to a customer. Note, that the salesperson could be honest and convinced about the utility of the item; nonetheless the situation is damaging. In education, the price we pay is students' time and suffering. And again, this happens often despite the good intention (and poor pedagogy) of teachers.

Mathematics built up a false image of a purely intellectual endeavor, thus it is usually perceived disconnected from life. Computing is in a better position in terms of motivation, as it is conspicuously pervasive in our everyday life. However, even topics in computer science may be losing their immunity to indifference. The success of software technologies may suggest that there are no problems to solve any more. For instance, explaining the PageRank algorithm to students born after Google requires depicting the age of Internet search where the relevant link was usually somewhere at the bottom of the page. Well functioning software tools could diminish the desire of understanding their underlying logic. The student has not experienced the problem, neither there is evidence that issue is important for society.

1.2 Creating motivating situations

Transmitting information in the 'teaching as telling' model of learning is inefficient without making sure that the audience is in the disposition of receiving. Prior to imparting knowledge, we have to create situations in the classroom, in which questions spontaneously arise, where students can face a real problem themselves. They need to meet a natural difficulty. Preferably it should be the same obstacle for everyone, in order to make group work and collaboration possible. Then, we can deploy methods for obtaining solutions; either just giving them away, or even better, leading the students to discovery. The question arises: *How to create motivating situations?*

1.3 Playing games

Playing games is an integral part of our culture (Caillois & Barash, 2001; Huizinga, 1949). It also evolved as a form of entertainment (Donovan, 2018). Games are mostly considered to be fun things to do, therefore education can leverage them by tapping into this natural willingness and propensity. Now we can ask our question more precisely. *What game can we use to motivate studying mathematics and computing?*

2 The remarkable properties of Go

For developing thinking skills by playing games, we have a wide range of choices. We can quickly narrow down to traditional strategy board games, if we require a wide spectrum of expertise, i.e. the game cannot be mastered by humans in a short time; and if we want to have games with a long history and cultural embeddings.

Chess and Go are often singled out for their purported educational benefits. However, it appears to be notoriously difficult to pin down the beneficial effects of playing exactly. It is argued that it is difficult to measure the long-term effects (for young players), and standardized school tests may not be good measure assessing the impact of playing these board games (Rowson, 2019).

These two games also stand out for being the driving challenges for the development of Artificial Intelligence. Chess was *the* application domain for the field since its beginnings (Ensmenger, 2012), and Go was the final grand challenge in pure skill games (Silver et al., 2016). AlphaGo's breakthrough caused a recent surge in wider scientific (Koch, 2016) and public interest in the game. Both Chess and Go are suitable for our purpose of building a course. Here we choose the game of Go that has some unique properties.

Go is a two-player, pure skill, and turn-based board game. The players put alternatingly black and white stones down on an initially empty grid.

The goal of the game is to surround more territory than the opponent. Enemy stones can be captured as well by fully encircling them.

2.1 Go is abstract and complex

Being *abstract* means that unnecessary details are removed. Something is reduced to its essence and it can be defined in a succinct way. Moreover, abstract implies being non-specific. Therefore, something abstract can be related to a wide range of other things.

The rules of Go can be described in a couple of sentences. Nothing from the rules can be omitted without destroying the game. Chess is also an abstract board game, but on a different level, keeping some details of the real world. It is tied to kings and their armies, which of course still leaves plenty of possibilities for connecting to real life (Kasparov, 2007). We could leave out some of its rules (e.g. not including the bishop), which would give a different, but still chess-like game.

Complexity comes from the interactions of the simple parts of a system (Mitchell, 2009). A complex phenomenon is interesting, since we cannot summarize it with a single idea, thus we cannot master it in one shot. In Go, complexity arises from the interaction patterns of the stones on the board.

Adding these two together, we conclude that Go is potentially connected to many interesting complex phenomena. This gives the opportunity: *insights gained in Go could be transferred to other fields of knowledge*. This is the single general argument for playing Go in educational settings.

As a concrete example, we can consider the incomprehensible combinatorial chaos of Go (Tromp & Farnebäck, 2007), and how it is related to a grand cosmological picture. Meaningful games by competent players in creative competition are exceptional sequences of board positions. Beginners also learn quickly to distinguish between a random position and the snapshot of a game. We 'live' in a tiny part of the vast possibilities of all possible legal board positions. This parallels how we are at home in the universe in a sense: only some very special configuration of material, e.g. the surface of a planet with a protective atmosphere is habitable for us. Random arrangement of particles does not provide suitable conditions for life, just like a random arrangement of stones makes no sense for us.

It is a cliché that Go is like life itself. A game is a smaller version of our struggle for survival and prosperity (You & Cho, 2018). Or, the history of human civilization can be conceptualized as a giant game, in which natural disasters are moves by a formidable opponent, but the consequences of our own actions often catch us too.

On the board the arrangements of stones build up the emergent structures we talk about when discussing the game: good and bad shapes, groups of stones with 'eyes'. Individual stones do not matter, only their relationships. This is exactly the basic tenet of category theory, the 'mathematics of mathematics' (Cheng, 2015). The primary interest is not the mathematical structures themselves, but their relations.

Also, the objects of our world are built up from combinations of elementary particles and atoms via the interactions between them. It is often remarked the number of positions on the full board is way bigger than the number of atoms in the universe. This comparison is unfair to the universe. The correct way would be using the number of all possible configurations of matter in the observable universe". Constructing any desired configuration of atoms, "transforming *anything into anything* that the laws of nature allows" (Deutsch, 2011) is the ultimate goal of engineering. On the Go board something similar can be realized. Theoretically, when two players cooperate in making a game as long as possible, a large fraction of the space of all legal positions can be visited (Tromp & Farnebäck, 2007).

Therefore, in a very abstract sense, the game is a model of the universe. This is a grandiose metaphor, which can be exploited both for sciences and for the game. It also fits into a long tradition of using the Go board to represent many things, like the four seasons, the stars in the sky. Its abstract nature allows the game to symbolize anything that is important in a given age. The distinction between order and randomness permeates several branches of science. It is a fundamental issue even when the uniqueness and finiteness of our universe are questioned (Tegmark, 2014).

2.2 Thinking is unavoidable in Go

An interesting observation about the game is that "It makes you think." (Shotwell & Long, 2012). This is a surprising statement, since by definition this is true for all pure skill games. There are a couple of reasons why emphasizing an obvious property makes sense.

1. Rote memorization has minimal effect, if any, on playing skills. This is even true for opening patterns, since the individual games differ

after a couple of moves; unlike in Chess, where building an opening repertoire is important.

2. Pretending to make thoughtful moves without thinking does not seem to be possible. A lapse in attention is sensed by the opponent immediately, and it is widely believed that a player's approximate strength can be judged by a couple of moves in a game.

3. The apparent seriousness of the game , which is difficult to pin down, could contribute too. Its culture, and aesthetics of the equipment, the time investment might be factors. It is a shared experience of players, that even casual games turn into serious ones.

When playing a game, some questions are inevitable. The immediate ones are about a particular game. *How do I make territory here? How should I protect my group?* Then there is reflection on playing and improving on a larger timescale. For instance, *How can one become a better player? Is there a sure winning strategy? What does it mean to be strong?*. We can rely on the appearance of these questions in the players' minds. Moreover, the answers in the context of artificial intelligence contain a fair amount of mathematical reasoning, most notably combinatorics, game theory and probability theory. This is an ideal setup to teach general problem solving heuristics (Pólya, 1945) in the context of the game (Egri-Nagy, 2011).

Therefore, the game is an ideal candidate to serve as a 'real-world' problem introduced in the classroom. As the rules are easy to learn, and it does not take too long to have a meaningful experience of elementary tactics and strategies, Go could give a shared background knowledge for everyone in the class. This does not imply that everyone has to be on the same playing level. The handicap system of Go can equalize the fight.

3 The positive role of Artificial Intelligence

It is hotly debated how AI technologies will change our lives for better or worse. Considering all possibilities is an immense task (Tegmark, 2017). Here we focus on some short-term benefits.

3.1 AI as a mirror

Thinking is one of our most important abilities. Therefore, improving it is also critical. How can we improve our thinking? We have to think about our thought processes, reflect on them.

The advance of AIs in Go could be viewed in many different ways (Egri-Nagy & Törmänen, 2020). For instance, losing the supremacy of human players can induce adverse reactions. However, some techniques are vindications of human thinking. They are often modeled after our thought processes. Logical thinking in solving a Go puzzle is made precise an systematic in classical search algorithms (Russell & Norvig, 2009). Intuition is modeled by the pattern recognition of neural networks. The training algorithms for deep learning networks justify the best human learning method: playing and replaying games.

On the other hand, randomized algorithms, like random playouts in Monte-Carlo tree search are not something a human player could do. We cannot track meaningless random moves in our head. However, the strength of the randomized algorithms is prompting us to develop a better sense for probability and statistics.

The engineers of AlphaGo found a way to integrate the wisdom of human masters into a convenient 'search engine' for the next move (Silver et al., 2016). This is putting the knowledge of all masters (all the game records, books, etc.) into a different container, an artificial neural network. Playing against AlphaGo is playing against all masters, not just a single opponent.

As the next step in the development of the software package, AlphaGo Zero could reconstruct and surpass all human wisdom in three days (Silver et al., 2017). It is a bit like that in Go we failed as a species to fully understand the game. However, thinking that we had already discovered everything that can be known about the game is overconfident. We tend to put ourselves into a privileged position, as a final goal of evolution. This is a mistake, which can be seen easily by following the history of our species (Harari, 2015).

In a way AIs provide a mirror for us. We can look into it and see ourselves: our logical thinking and intuition, and their limitations. Or, we can see our improved selves. The AIs can also give guidance on how to improve our thinking.

3.2 AIs as democratizing force

Beyond teaching at the undergraduate level, another beneficial use of AI go engines is that it makes learning the game easier for everyone. There is always a strong player ready to play. Moreover, with the advance of analysis tools, now everyone has a strong player to review a game with. Like the printing press, knowledge is more democratically distributed, allowing everyone to enjoy the game more. The same happened in the world of chess (Kasparov, 2017).

It is more important to provide access, not just to the game, but for the AI technologies themselves. This is an important role of university courses.

4 Knowledge transfer

Knowledge transfer is the hallmark of successful learning (Barnett & Ceci, 2002). The question is how exactly this knowledge transfer from the game of Go can be done or facilitated. We envisage a couple of ways.

1. Directly related courses, such as Mathematics, Statistics, Programming, and Machine Learning, could benefit from a thoroughly discussed example.

2. Courses discussing the societal and political changes induced by the advance of AI technologies could be better understood by a clear understanding of these technologies' core concepts.

3. Studying, in general, could benefit from the experience of improving Go playing skills. Especially for beginners, practicing Go puzzles leads to quick improvement. Similarly, repeated exposure to fundamental ideas, instead of cramming the night before the exam, is essential in learning any subject. Of course, this connection is subtle enough that it requires explicit mentioning in the class.

4. The most speculative possibility for knowledge transfer is about life skills. Go is a game about finding the right balance between attacking and defending, between taking territory and letting the opponent live at some parts of the board. Managing life also requires the ability to find balance. For example, between study and social activities, between work and family, and so on.

Transferring skills between Go and mathematics or computer programming is not a straightforward process. As pointed out in Lee, 2016, professional Go players often know little about Mathematics since their education was focused solely on the game from early on. The conclusion is clear: to play well, there is no need for Mathematics. While in Europe and North America, where there used to be no professional systems, Go players were typically mathematicians or software engineers, hinting that these might be related skills. Outside Asia, people are often introduced to the game during their studies at university. However, these observations provide little insights into the possible connection.

In any case, the transfer between the game of Go and Mathematics cannot be direct. Mathematics is a symbolic language, and Go is not. However, when we look at the thought processes involved in both fields similarities arise. The expertise is built by transitioning through practice from a conscious step-by-step calculation process to a more automated pattern recognition ability. Similar to solving a Go puzzle (tsumego) by figuring out what move to choose next, the simplification of a logarithmic expression in algebra is about finding the right next move, i.e. choosing the appropriate law of logarithm. In both cases the trick is to choose suitable action from a set of possibilities – which could be a broad definition of computational thinking.

To improve this decision making process, the interaction between calculation and intuition can be improved by being aware of their capabilities (Kahneman, 2011). Therefore, we suggest that the transfer could happen on the level of metacognition. That is, the need for improving when playing games enforces self-monitoring. Then, this can be transferred to mathematical problem solving, where it is generally thought to be beneficial (Schneider & Artelt, 2010). *Metacognition* is the defining core of classical heuristics (Pólya, 1945). The strong game review culture of Go (You & Cho, 2018) is an implementation of these principles. An aspiring Go player frequently goes through both won and lost games to find mistakes and ways to improve. However, for a beginner player this might not be an obvious action to take. To promote self-reflection the course will contain writing (about the effectiveness of the chosen study methods and the comparison between natural and artificial intelligence) and oral presentation components (game reviews and Go puzzles). The reflective thought process could increase the probability of knowledge transfer as well.

5 Summary

We reviewed the potential benefits of using the game of Go in an undergraduate course. Based on cultural and educational considerations, we conclude that using an ancient game is a valid approach for tackling some current issues in Liberal Arts education. This theoretical analysis will be followed by an empirical investigation of a course implemented according to these guidelines.

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