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Igo Math Natural and Artificial Intelligence and the Game of Go

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These preliminary notes are being written for the MAT230 course at Akita International University in Japan. Some parts are finished, others just barely started. Comments are welcome! When reporting errors please specify the version number. The latest version can be downloaded from `https://egri-nagy.github.io/igomath/`

Contents

What is this book about?

What is contained, and what can we gain from reading this book?

This book is about *the game of Go*. It describes the rules and teaches some elementary tactics and strategy. However, it does not contain a training program and it does not introduce the culture of the game. Therefore, it cannot compete with decent introductory books.

The book is also about *artificial intelligence*. It describes its core ideas and fundamental algorithms. It explains how inanimate 'mechanisms' can imitate and surpass the abilities of our thought processes. However, there are more detailed and more comprehensive textbooks on this technical field.

Once we talk about something technical, a precise language is needed. By definition, that is mathematics. Obviously, one can build up Go strength without studying mathematics (as all professional players do). But, if we want to talk about that expertise, or want to explain to a computer how to play the game, logical exactness and statistical analysis are needed. Therefore, we will introduce concepts from combinatorics, graph theory and probability theory. We will restrict the mathematical content to those concepts only that are needed for understanding the game. Again, there are better textbooks for these mathematical topics.

Why this book then? What is the problem we are trying to solve here? Consider the following scenario. A student's meeting with her academic advisor.

STUDENT: I worked hard, but my results are not good. I have to improve.

ADVISOR: What is the problem? Do you have a plan to fix it?

STUDENT: I don't know, but I will work harder.

What is wrong here? The student has an evidently failing method for studying. With good intentions, the idea of doubling the effort comes naturally. However, a good advisor should say something like this.

ADVISOR: Maybe, you need to work 'easier'.

Mental effort is tiring, so the wrong method will make more trouble, failure guaranteed. The student needs to find a different way for studying, a more efficient method. For that purpose, one has to perform self-reflection. Asking questions like *How do I study?*, *Can*

I do it more efficiently, maybe in shorter time?, *What did I do before successes/failures?*, etc.. This is not easy. Self-reflection is a learned skill.

The main purpose of this book is to facilitate the reader to learn more about his or her thinking skills. Go, as any other abstract strategy board game is a clean laboratory for experimenting with our minds. *What is my strategy? Is there a better one? What did I do in won/lost games?* These questions are similar to the questions about the learning method above, but easier to answer. Go is lot simpler than life itself. We can always play another game, in which we can use our knowledge gained from previous games. In life, we have to come up with a good move in each new situation, and often there is no way to repeat the situation, or at great price (e.g. choosing partner, a profession, schools, etc.). Real world problems are way more complicated than board games, but wisdom gained on the board could be transferred to life. This is the main assumption and the promise of this book.

AIs are often modelled after our thinking processes, therefore now they can serve as mirrors, in which we can see ourselves. By studying AI algorithms, we can understand our thought processes better, we can get a new appreciation of the capabilities of the human brain.

We can summarize these points in an 'equation':

$$\frac{1}{3}\text{ Go} + \frac{1}{3}\text{ AI} + \frac{1}{3}\text{ Mathematics} = \text{transferable metacognition skills.}$$

We don't know what skills future jobs will require. It is a reasonable guess that the one will need to learn new skills often, which is demanding. Therefore, training emotional intelligence and for mental resilience is a good investment.

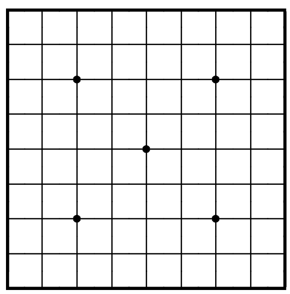
The rules of Go for human beings

How to play Go? What are the rules?

Board and Stones

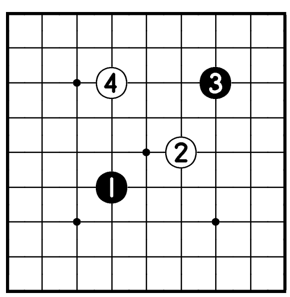
The game is played on a square grid, like this.

The size of the grid can be different. 9 × 9 is a good start for beginners, allowing quick tactical games, 13 × 13 requires deeper strategy, while 19 × 19 is the standard size.



The little dots on some intersections are there only for finding our way on the board.

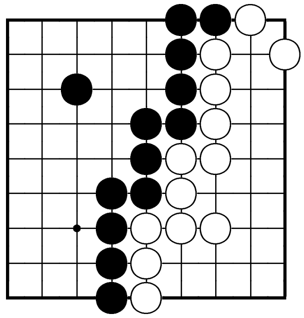
Black and white alternates in making moves by placing stones on the empty intersections. Here are the first four moves of a game.



Once the stones are placed, they don't move.

Surrounding territory

The goal is to surround territory. In order to win one has to have more territory than the opponent. Here is the result of a peaceful game.



Even without precise counting, one can see that black has more territory on the left than white on the right side. Therefore, black wins the game.

However, games are not always this peaceful. There might be clashing territorial claims, decided by fighting. In Go this means surrounding groups of enemy stones.

One puzzling question for beginners is to decide the ownership of a territory. For instance, “There are invading black stones in my white territory. Is the territory still mine?”. The answer depends on whether the invading stones can form an indestructible territory inside, or not. This can be decided by surrounding the invading stones.

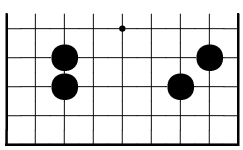
Chains and liberties

Stones form *chains* when they are connected either horizontally or vertically. A chain is an unbreakable unit. In terms of surrounding a chain is a single thing; once a stone is in a chain it cannot be surrounded as an individual stone.

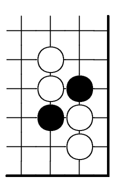
The word ‘chain’ is used metaphorically. It’s about the connectedness of stones of the same color, not about having a long, signle-threaded shape.

Here are a chain of two stones (left) and two disconnected stones (right). There is no diagonal connection, just as there are no diagonal lines on the board.

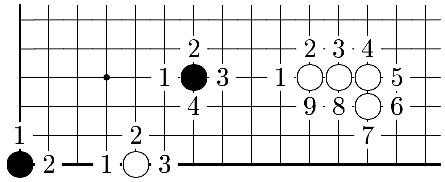
A stone in itself can be considered as a chain of size one.



The diagonal black stones can be separated by white stones.

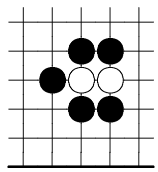


The chains need “breathing space”, i.e. empty intersections in direct contact with stones in the chain. These are called *liberties*. The number of liberties is an important property of a chain, and counting liberties is crucial in tactical fights. Here four chains with their liberties counted.



Surrounding a chain is reducing its liberties. A special situation is when a chain has only one liberty left, and we say it is in *atari*.

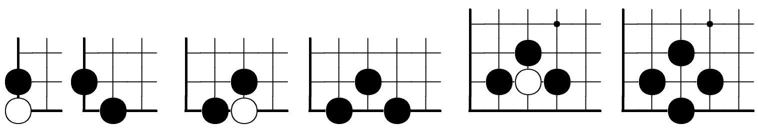
Is it possible to be in atari without contacting an enemy stone?



White is in atari, it has only one empty neighbouring intersection.

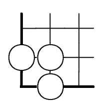
Capturing

Capturing a chain is filling its last remaining liberty, when all stones in that chain are removed from the board and they are kept separately as prisoners. Here are three examples of capturing.

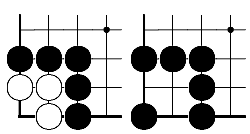


Self-capturing is not allowed. Black cannot make a move into the corner as that intersection is surrounded by white.

Prohibiting self-capture is not necessary. Some rulesets allow it. While it may look useless, it can make a difference in ko fights (see later).

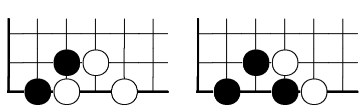


Though one can fill his/her own last liberty in order to capture an enemy chain at the same time, which creates new liberties, so the move is not a self-capture anyway.



Avoiding eternal games

Another illegal move is the one that would repeat a previous arrangement on the board.



Black captures the white stone, and while the capturing stone ends up in atari, white cannot capture as that would restore the previous situation. White is obliged to play somewhere else first. This is the *ko rule*. In addition to ensuring the finiteness of games, it leads to an interesting game dynamics by involving remote parts of the board in the same fight.

Scoring

A game ends in agreement, with two consecutive passes, when none of the players can, or want to make further moves. This happens when for both players the status of each chain on the board is clear (whether it is not possible capture, or it is not possible to save it from capture).

In the beginning, deciding whether a game is finished or not, is not at all straightforward. As a quick rule of thumb, if the territories are not sealed off, then the game is not finished yet.

One way to find the score is counting *territory*, the surrounded empty intersections, and then subtracting both the number of captured stones and the number of stones get caught in enemy territory and not able to surround territory on their own.

How many points is a captured stone worth?

Alternatively, one can count *area* the surrounded empty intersections together with the friendly stones on the board.

The score for white is increased for white by *komi*, which is an agreed amount (5.5, 6.5 or 7.5) to offset the advantage of black moving first.

How do we know that black has an advantage? Game records showed an imbalance between the winning frequencies of black and white. These values are set by the statistical analysis of game record databases. Half a point is added as a tiebreaker.

Starting to play

These rules describe what are the legal moves in a game, so after reading these one can start to play valid Go games. However, they give no instructions on what are the good moves. Mastering the game is a long and gradual process. For beginners, there is only one advice. *PLAY!*

In the first few games it is important to simply observe what happens on the board with an open mind, without trying too hard to win. If one insists of having at least some guidelines even in the very beginning, then here is a simple and easy to remember strategy.

> "... if you see an enemy stone, try to capture it, or cut it off.
> If you see a friendly stone, try to save it from capture, try to connect it." [1]

[1] T. Kageyama. *Lessons in the Fundamentals of Go*. Beginner and Elementary go Books Series. Kiseido Publishing Company, 1998

The logical rules of Go – for computers

How to 'explain' Go to a computer? Can we describe the rules with mathematical precision?

One thing is to roughly explain the rules for human beings, so they can start playing right away. If something is not fully clear, they can pause, discuss the matter and then continue the game with some mutual agreement. Humans are good at dealing with unclear situations cooperatively.

It is a different level of precision if we want computer to play the game. The rules have to be strictly watertight and unambiguous. The computer cannot stop and negotiate. A Go playing program is a mechanism after all. At each step it has to be clear what to do next.

As such, they are not really suitable for human consumption. Giving only mathematical rules to beginners would probably reduce the number of players worldwide.

Here are the rules that are more clear logically (based on the Tromp-Taylor rules [2]), and can be used for implementing the rules in a computer program. The comments can shed light on the meaning of the terse mathematical description and show the connections with the intuitive rules.

[2] J. Tromp. Tromp–Taylor rules. `http://tromp.github.io/go.html`, 1995

1. Go is played on a $m \times n$ grid of points, by two players called Black and White.

   Traditionally square grids, but nothing in the rules rely on m being the same as n. In fact, the game can be generalized to be played on any set of points with an adjacency relationship (for each point we can find its immediate neighbours, mathematically speaking this structure is called a *graph*).

2. Each intersection point on the grid may be colored black, white or empty.

   The term 'coloring' naturally describes the act of putting down a stone (while removing a stone may be understood as coloring the intersection empty). The coloring idea also comes from *graph theory*, the mathematical study of relations between objects.

3. A point P, not colored C, is said to *reach* C, if there is a path of (vertically or horizontally) adjacent points of P's color from P to a point of color C.

   This idea of reaching, or seeing, or touching is the crucial one for defining a simple rule set.

   This compresses a lot of meaning into a single rule. It implicitly defines the chains (points of the same color connected by path(s) along the grid lines). Then, for a chain, we check what *other* color(s) does it have contact with. Interestingly, this is done from the perspective of a single point. The chain emerges due to the fact that all points in a chain reach the same set of colors. A point does not reach its own color.

4. *Clearing* a color is the process of emptying all points of that color that don't reach empty.

    This process ensures that there will be no chains left on the board with no liberties. The empty color has the role of deciding what stones can stay on the board.

5. Starting with an empty grid, the players alternate turns, starting with Black.

    Alternating turns are common ot many strategy board games. Starting with black is traditional.

6. A *turn* is either a pass; or a move that doesn't repeat an earlier grid coloring.

    Turn is a word for covering two different possible actions. Passing is doing nothing, the position on the board does not change. It happens when a player thinks the game is over. A move changes the position, but cannot go back to a previous position (positional superko).

7. A move consists of coloring an empty point one's own color; then clearing the opponent color, and then clearing one's own color.

    A move is a three-stage process. 1. placing a stone, 2. capturing enemy stones, 3. capturing friendly stones. The order is important and there are some logical relationships between stages 2 and 3: at most one of them can happen in a move. If 2 happens, 3 will not take place: capturing enemy stones will guarantee at least one liberty for the capturing stone. If 2 does not happen, 3 might happen: the case of self-capture, suicide move.

8. The game ends after two consecutive passes.

    Dead stones are removed from the board and scoring begins. If there is a disagreement about the status of some chains, playing can resume.

9. A player's score is the number of points of her color, plus the number of empty points that reach only her color.

    This is are scoring. It allows to fill up one's own territory, since it doesn't matter whether we count a point as a surrounded territory or as a friendly stone. Thus proving that enemy stones in one's territory can be captured has no cost.

10. The player with the higher score at the end of the game is the winner. Equal scores result in a tie.

    Komi (points added to white) can be used for adjusting the scores to offset black's advantage due to starting first. Komi including half a point breaks ties. The exact value of komi is disputed, depends on statistical evidence only.

Time control

The ko rule ensures that a Go game will end in a finite amount of time. However, for practical purposes, such as tournaments or classroom activities, this is not enough. We need to predict the length of a game as well, not just the fact that it will end. Also, good moves need time consuming consideration, therefore time limits make the game harder to play well. Faster games more rely on intuition than calculation skills.

The basic idea of time control is to penalize using too much time. One can even loose a game due to the lack of time. There are numerous ways to control time. The different methods can be classified by the following questions.

1. *Do we limit the total, or the time for making a move, or both?*
2. *Is the available time fixed, or changes dynamically?*
3. *How to manage overtime? What happens when a player has no time left? Is it immediate loss, or is the player allowed to play under more severe restrictions?*

Here are some frequently used time control methods. The most basic ones are those where going overtime means loosing the game, regardless of the board position.

Absolute A fixed amount of time is given for the whole game. It's up to the player how much time to spend on each move.

Simple A fixed amount of time is given to each move. The length of a game can be estimated by the average number moves in games.

In more sophisticated methods, going overtime leads to some 'grace period'. The player can continue but on different terms for time. Thus depleting main time is not an immediate loss, but one is forced to play faster.

Byo-yomi (countdown) After depleting the fixed main time, the player has a fixed number of time periods. If a move is made within a period, then it restarts for the next move. Otherwise, it expires, reducing the number of available time periods.

Canadian After the main time, the player has to make a certain number of moves within a time period in order to get another time period.

Then there are schemes where the available time changes dynamically, rewarding moves.

Fischer (bonus) Initial time is given to the player, then each move earns bonus time. The bonus times can be accumulated to a certain limit.

Rankings and ratings

What does it mean to be good at playing Go? How can we measure playing strength? Why do we need to measure strength?

Ranking is about comparing players. Telling whether one player is ranked higher or lower than the other. *Rating* is putting a player on a common scale, no comparison is involved. In everyday usage, these terms are not distinguished strictly.

Traditional Go rankings

Student ranks are *kyu*s. Beginners start at 30kyu and by playing a couple of games quickly advance to around 20kyu. Casual players (19-10kyu) are often referred as DDKs (double digit kyus). Similarly, intermediate amateurs are called SDKs (single digit kyus). After 1kyu one can reach 1dan – the equivalent of the black belt. After that levels can go up to 7dan.

The idea of these ranks is that difference between the ranks determines how many handicap stones should be given to the weaker player in order to have an even game.

The handicap stone calculation is valid on the 19×19 board. Playing black with no komi or 0.5 is one handicap stone.

Professional players have a different scale from 1dan to 9dan. The levels closer to each other than a full handicap stone.

Élő rating system

The idea is that player's rating should predict the probability of winning a game.

Here is the notation used.

players	A, B
player ratings	R_A, R_B
expected scores	E_A, E_B
game result	S_A, S_B

Rating values starting from 2300 indicate master level play. A game result can be 0 (loss), 0.5 (draw), and 1 (win). The expected scores are the probabilities of winning, therefore $E_A + E_B = 1$.

$$E_A = \frac{1}{1 + 10^{\frac{R_B - R_A}{400}}}$$

$$E_B = \frac{1}{1 + 10^{\frac{R_A - R_B}{400}}}$$

The computation can be simplified by letting $Q_A = 10^{\frac{R_A}{400}}$, and $Q_B = 10^{\frac{R_B}{400}}$. Then

$$E_A = \frac{Q_A}{Q_A + Q_B}, \; E_B = \frac{Q_B}{Q_A + Q_B}.$$

How does this simplification work? In the exponent we can write the fractions as two separate fractions.

$$E_A = \frac{1}{1 + 10^{\frac{R_B}{400} - \frac{R_A}{400}}}$$

Then using the law of negative exponents,

$$E_A = \frac{1}{1 + \frac{10^{\frac{R_B}{400}}}{10^{\frac{R_A}{400}}}},$$

so

$$E_A = \frac{1}{1 + \frac{Q_B}{Q_A}}.$$

After a game result S_A, S_B (note that also $S_A + S_B = 1$) the updated rating can be calculated by

$$R'_A = R_A + K(S_A - E_A),$$

where K is the so called K-factor regulating the speed of rating changes. It is bigger for lower rated players, for instance $K = 32$ and smaller for master players, like $K = 16$.

Improvements: Glicko and Glicko-2

`http://www.glicko.net/glicko.html`

Ratings reliability. *RD*, ratings deviation (1 standard deviation), measures the accuracy of a player's rating.

Rating volatility, the expected rate of fluctuations in one's rating. The measure of how consistent is someone's performance.

Building up terminology

How can we say it in simple words, what is happening on the board?

Whenever we want to talk about something, we need to find words for describing the object. We can create new words, or assign new shades of meaning to existing words, or used them metaphorically. Mathematics is a way of building a language precisely, often not by words, but simply by symbols. First, we just need a descriptive and intuitive language for talking about what happens on the board.

People often rely on intuitive understanding, communicating based on the given context. Therefore, few books get strict about Go terminology, see for instance [3] – written by a software engineer. Stones connected or close to each other are often referred as 'groups'. Here we refine this concept by identifying the 'building blocks' of board positions.

[3] B. Wilcox and S. Wilcox. *EZ-go: Oriental Strategy in a Nutshell*. Ki Press, 1996

Stones are the smallest units, the 'atoms' for building a game.

Chains are formed by stones that are in direct contact vertically or horizontally.

Links are close range, but not direct connections.

Groups are links of chains.

Two eyes

Rules say nothing about shapes with two eyes being unconditionally alive. It is an *emergent* property.

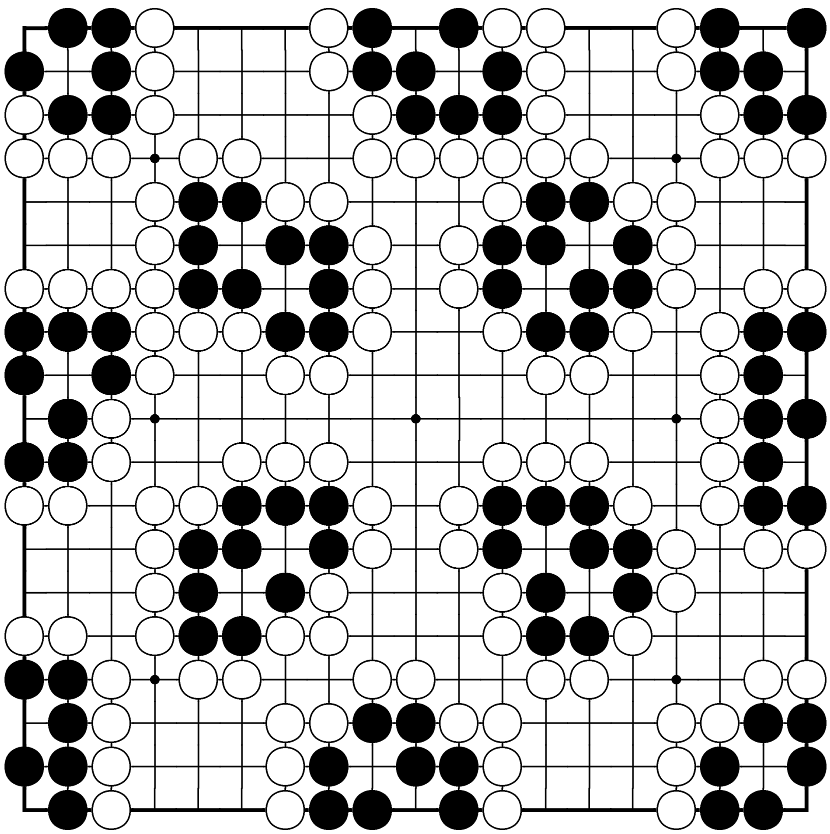


Figure 1: A collection of minimal living shapes.

Activity: Liberty Analysis

Counting liberties during the game is important, but beginners often neglect to do so. They recognize atari, but that might be too late. Here is an exercise that may help to see the dynamics of liberty counts for chains on the board.

Take a 9×9 game record and replay the game. For each move,

- for isolated stones, record its coordinates and number of liberties – a new chain is created
- when attaching to an existing chain, update the liberties of the existing chain by simply writing down the new number (so the history of liberty changes can be seen); if it is the same number, then it may be omitted
- when two (or more) chains get connected, keep the most recent one with recomputed liberties and cross out the previous one(s);
- in any case, update the liberties of opponent chains

This list is similar to data structures that classical AI Go playing programs use to keep track of the chains.

Game Trees

How to represent a Go game mathematically?

A game proceeds by moves, from a board position to the another one. A game is a sequence of board positions, therefore the 'things' we are studying are these arrangements of stones. We also have a relationship between them. A position is a neighbour of another one if a single move can lead there. Note that this relationship is one-way. There is no guarantee that we can go back to the previous position.

Having some objects (board positions) and relationship between them (defined by making moves) hints a graph structure. What sort of graph? Looking at an actual game, it is just a sequence: a succession of board positions from the empty board to the final arrangement. It looks like a single line, but this does not capture a very important aspect of the game. Namely, a game is a decision based process. It is not just about the actually played moves, but about the unplayed ones as well.

The statement that unplayed moves matter a lot in the game is not at all paradoxical. Only total beginners play the game by move to move. An advancing player has to read ahead and consider variations, and possible threats.

What is a decision? It is a choice between possible actions. Here, a choice between possible moves, and thus between future board positions.

Bibliography

[1] T. Kageyama. *Lessons in the Fundamentals of Go*. Beginner and Elementary go Books Series. Kiseido Publishing Company, 1998.

[2] J. Tromp. Tromp–Taylor rules. `http://tromp.github.io/go.html`, 1995.

[3] B. Wilcox and S. Wilcox. *EZ-go: Oriental Strategy in a Nutshell*. Ki Press, 1996.