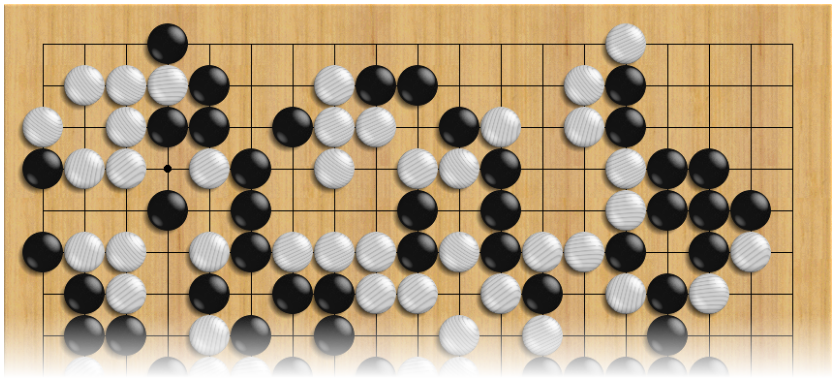
"The rules of Go are so elegant, organic, and rigorously logical that if intelligent life forms exist elsewhere in the universe, they almost certainly play Go."
Edward Lasker, chess master



MAT 230
AI, Games and Mathematics: Igo Math

Natural and Artificial Intelligence and the Game of Go

Level:	200
Credits:	3
Web:	https://egri-nagy.github.io/igomath/
Instructor:	Attila Egri-Nagy
Office:	A3-4
Email:	egri-nagy@aiu.ac.jp

v2020.01.07

Highlights

"batteries included" The course includes a complete introduction to the game and regular matches on real equipment in the classroom.

no stress Playing strength is not part of the assessment. Exams contribute only 40% to the final grade, the rest comes from classroom activities, homework problems and writing assignments.

useful mathematics Mathematical content is only introduced for answering real questions, which arise from playing the game and from trying to understand the thought processes involved.

meet the AIs Neither living, nor conscious. Still, far beyond us. Better to know them.

bottom line It is all about your thinking skills and emotional intelligence.

Course Description

Go is an abstract board game of superlatives. (Chinese: 围棋 weiqi, Korean: 바둑 baduk, Japanese: 囲碁 igo). Most ancient (originated 3000-4000 years ago), most abstract (minimal set of rules, single aim of gaining territory), most complex (extreme high number of patterns, variations and game plays), most peaceful (winning is by gaining more territory, not by annihilation; games end in agreement), most beautiful (black and white stones on a grid laid over the natural grains of wood). It is one of the highest art form of logical and intuitive thinking combined. Renewed interest in the game was sparked by a somewhat unexpected event: a computer program won against the world champion player in 2016.

How can machine intelligence surpass human intellect? How do we think when we play Go? Do we now have a complete understanding of the game of Go? The course will answer these questions by studying the game and its computational aspects by using mathematics as a tool.

Learning Outcomes

On the successful completion of this course, it is expected that the students will be able to:

1. play the game and solve elementary Go problems (tsumegos)[1], appreciate its aesthetics and its rich cultural heritage;
2. identify the cognitive processes involved in playing, and understand the difference between amateur and professional players;
3. represent games as tree structures, and solve simple games by applying the minimax search algorithm;
4. use techniques of combinatorics to enumerate all possible plays for simple games, understand the role of symmetry as a mathematical tool;
5. sense the difference between computationally hard and easy problems;
6. understand probabilistic sampling techniques and Monte-Carlo simulations;
7. grasp the ideas of machine learning and neural networks, and assess their impacts.

To single out the most important benefit of the course, it is expected that students will be able **think about their own thinking process**. This universal skill is likely to have lasting impact on the students' performance in all other fields of study.

Notes on the level of difficulty

Scholarly investigation of a game requires mathematical and computational formalism. Fortunately, for the game of Go, the requirements are not some advanced material, but 'low-hanging fruits' of several parts of mathematics (e.g. graph theory, probability, algebra, combinatorics) and artificial intelligence concepts (e.g. game trees, minimax algorithm, neural networks). These will be explained from first principles, making the course enjoyable for all students.

[1] This course is a scholarly activity, which happens to have a component allocated for playing. Experiencing the game is a necessary requirement for studying it. However, the course does not guarantee that students will become strong players.

Tentative Schedule

The schedule of the course is designed around motivating questions that naturally arise when playing the game.

Week	Topics
1	**Introduction.** *What is Go?* The rules of Go. *How was it invented/discovered?* History. *How to play Go?* Tactics and strategy.
2	**Learning and improving.** *How can one become a better player? What do professional players do?* Go problems (tsumegos). Metacognition. Go proverbs.
3	**Ratings and rankings.** *How to measure progress in learning? How to rank players?* Traditional kyu and dan system, winning probabilities and the handicap system. Élő and Glicko rating systems. Elimination, ladder and all-play-all tournaments.
4	**Game tree representation.** *How to describe gameplays in a precise manner?* AI concepts: search space and evaluation function. Graph theory. Tree structure to linear text – the SGF file format.
5	**Minimax algorithm.** *How to solve a game?* Different notions of a solved game. Search algorithms in classical AI. Heuristics. Adversarial search.
6	**Computational complexity.** *What makes a problem difficult for a computer? What are combinatorial explosions?* Polynomial and exponential growth rates of execution times of programs. The "horizon effect" of game tree search.
7	**Enumerative combinatorics.** *What is the size of the game tree? How many legal positions are there?*
8	**Symmetry as a compression tool.** Shapes on the board rotated and reflected, switching colors.
9	**MIDTERM TEST**
10	**Statistical methods and probabilistic sampling.** Randomness as a tool for dealing with hard problems. Monte Carlo simulations. Exploitation versus exploration. The multi-armed bandit problem.
11	**Architecture of a Go playing program.** *How to evaluate a board position? How the choose the next move?* The classical pattern-matching algorithm of GnuGo.
12	**Neural networks.** The inner workings of the AlphaGo system. AlphaGo Zero, reinforcement learning. **The impact of AlphaGo.** *What happens next?* Responses by Go players. Parallels to the more general phenomenon of automation.
13	**Class tournament.** Double elimination tournament. Game analysis of the final match. **13×13 games.** Introducing concepts for the opening. Josekis.
14	**Variants of Go.** *Why the rules of Go are so special?* The notion of emergence. Exploring the 'space' of games in the neighbourhood of Go. Irregular Go boards, Go in higher dimensions, more than two players.
15	**FINAL EXAM**

Assessment Components

First of all, though playing skills are continuously assessed in this course, **strength does not directly influence the final grade**. Therefore a beginner can have an excellent grade, while dan level players are not guaranteed to have top results. The assessment design aims to reward the scholarly effort. Secondly, unlike a traditional mathematics course, here we put less emphasis on the exams (they only contribute 40% to the final grade). Instead, we prioritize **reflecting on thought processes** (the writing component) and **sharing ideas** (discussion and oral presentation parts).

Writing assignments 15% ($3 \times 5\%$) Three short essays helped by guiding questions, forming a growth diary for the semester. These are written records of events and stages of understanding the game and its theory and the relationships with the AIs.

Classroom activities 20% Continuous evaluation of participation in gameplays and paired problem solving. Writing and analyzing game records. Emphasis will be put on sharing thought processes, explanations and helping others will be rewarded. This implies that **participation is mandatory**. During winter semester 2 days can be missed without any effect on the grade. Starting with the third day, each absence is worth 4 grade points until the 20 grade points are depleted. Similarly for spring and fall semesters, 3 classes can be missed then each absence is worth 2 grade points. Missed class is defined by missed game. Roughly speaking, missing about half of the classes means that best achievable grade is C.

Presentation 10% [2] Short presentation in the classroom: reviewing a game and explaining an interesting Go problem.

Homework assignments 15% ($5 \times 3\%$) Selected elementary mathematical problems, relevant to the game of Go, coming from combinatorics, probability and graph theory.

Midterm & final exams 40% ($2 \times 20\%$) Paper based exams containing problems similar to homework assignments and basic questions about the game itself.

Assessment details

Writing: short essays

The course requires three 500 word essays, submitted online. One in the beginning of the course, one in the middle and one at the end of the semester. The marking is all or nothing, however delayed submissions will incur a 10% reduction in score and partial submission will be scored pro rata. In case of poor grammar and spelling mistakes the essay may need to be corrected and resubmitted. Uncorrected essays will earn no grade points. The best way to write the essays is to think about the questions below and answer them **as honestly as possible**. Sample essays are also provided.

Igo and Me *Did you know the game before starting this course?* If yes, *When did you start? Who taught you? Where did you play? How often?* If no, *Do you play some other games? Which ones? Did you hear about Go before? What was the reason for not trying the game? Why did you decide to take this course? What do you expect from studying the game?*

[2] There is a possibility of replacing the presentation with another activity, e.g. reading a relevant Japanese book and creating an English summary. These require more work, and can be arranged by discussing it with the instructor.

AIs and Me *What programs did you play against? Did you recognize any style in those games? How does it feel to play against the computer? Which one is better, winning against a human opponent or against the machine? How about loosing? Does it change your experience if you know how the Go program works? What do you think about the future of Go playing in the presence of superhuman AI Go engines?*

My learning method and reflections *What was your approach for getting better? Did you just play, or did you have a more systematic method? Did you have to change your method?* Give details! *Do you think your method was successful? How much stronger did you get? Was there a moment when you felt leveling up, a clear sense of progress? Did you discuss your method with someone? Did you compare your ways of studying with others? What do you think of the game now? How did your opinion change during the course? Are you planning to play again? Do you think your thinking skills in general benefited from studying the game?*

Presentation: game records

Commented SGF files should be submitted online. Prior to the presentation, **they have to be discussed with the instructor**, where the games will also get an AI analysis. The Go puzzles can be chosen freely (with considering the level of the class), or upon request they will be assigned by the instructor. In general, the same game cannot be reviewed by two students. However, some outstanding games might be excepted.

Game review, commentary Using a game record editing software, write commentary for your best, or most interesting game. Give information about the game, *Who/What was the opponent? How strong? Where was it played? When? Under what circumstances?* Provide game information: komi, handicap, time control. Describe the overall character of the game, *Was it a peaceful one? Did it have some fierce fight?* For each of your moves (except the most obvious ones, like endgame moves), consider the following questions. *What was the purpose of this move? What did I think? What did it achieve?* Also, for the opponent's moves, *Was the opponent's move surprising? Did it change your plans?* Regarding the whole game, *What was the deciding move? Who had the initiative?*

Tsumego Given a Go problem, find the solution(s) and comment all meaningful variations. *Which one is the solution, and why? What was the main idea? Are there other solutions? How do they compare?* It is important to explore the failing variations too.

Presentation guidelines

The main purpose of the presentation is knowledge sharing. Other students can learn from your game and problem solving. Therefore, an ideal presentation is motivated by the following question. *What is the experience, insight that I can share with other students?* Common mistake is to treat the presentation merely as a grade point collecting duty, and thus missing the opportunity for an enjoyable and educating social event.

Recommended length is 5 minutes. The presentation can be done by using game editing software and the projector, or the physical demonstration board. For tsumegos, the best style is to ask the students to set up the problem on the Go boards and interact with their solution attempts. Here are some further guidelines for presentations.

1. **Give the audience some time to understand the situation.** In game reviews, go through the opening moves slowly. They often determine the final shape of the game. This

is a bit like introducing the characters in a story. In tsumego presentations, state the goal of the problem clearly, e.g. black to play to live in the corner, white connect, etc.. For those, who solved the problem before, few seconds are enough to solve it again, and they can benefit a lot from that.

2. **Tell a story!** In a presentation, the narrative is the added value to the game record (currently AIs don't do that). Hence, telling the result in the beginning may not be the best option.

3. **Quality is more important than quantity.** No need to cover many variations, it's enough to pick a decisive one. You can click through variations lot quicker than the audience can grasp them. It is more rewarding to spend a bit of time on a single question. For tsumegos, you should assume perfect play from the adversarial player. This can reduce the number of variations.

4. **Know your tools!** All software tools have their quirks and useful features. For example, Sabaki has useful keyboard shortcuts (e.g. try to navigate a game tree with the arrow keys). It takes only a couple of minutes to get to know these tricks, and it makes a presentation lot easier. The audience will be thankful for a fluid presentation.

5. **Know the material!** During a presentation, you only have half of your attention and memory available (the other half is busy with the situation). Therefore, it is important that you know the game and problem well, so you could present them convincingly. You don't have to memorize them (btw., that's an excellent exercise), but going through a couple of times is highly recommended. Otherwise, you will end up clicking through the moves quickly, back and forth, just to find the move you want to talk about. Impossible to follow for the audience.

6. **Details do matter.** Don't forget to put the names of the players in the SGF files. Someone might miss when you say that in the beginning, and will spend the rest of the presentation guessing whether your are black or white.

Midterm and final exams

The exams are in the traditional written format. Problems are similar to the ones given as homework assignments. Sample tests will be provided.

Go related problems

The problems here are very fundamental questions about the game, and not for measuring the strength of playing skills.

- Identifying and counting chains.
- Deciding the status of a group (e.g. recognizing false eyes, seki, and ko situations).
- Counting scores on a finished game on a small board.

Probability Theory

- Using Élő's algorithm, given two ranked players, how should we adjust their ranks based on the outcome of a match?

Graph Theory

– Recognizing types of graphs (directed, tree, tournament, planar).

Combinatorics

– Calculate the number of possible arrangements on a game board. Calculate the growth of game trees.

Artificial Intelligence

– Solving a simple search problem by drawing the search graph.

– Drawing the game tree of a simple game.

– Executing the minimax algorithm on an abstract game tree.

– Calculating the activation of an artificial neuron cell.

Delivery Format

A balanced combination of playing games (free and instructed), solving problems (guided and independent) and lectures. Go equipment is available in classroom. **Class capacity:** 24 (even number preferred).

Requirements

Knowing how to play the game is *not* a requirement, since the basic rules can be learned in 5 minutes. The course is self-contained, as the mathematics part of is built up from first principles. Therefore, minimal math background, like regular high school math, suffices.

A **scientific calculator** is needed for the exams. The calculator should be capable compute powers of 10 with fractional exponents. Phones cannot be used during the tests.

Textbooks

There are many excellent books about Go, but only a few (and rather technical) research monographs on the mathematics of Go. There is no established textbook about the mathematics of Go. Therefore, lecture notes and handouts will be provided.

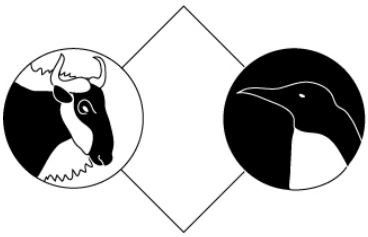
Software & Resources

Playing online

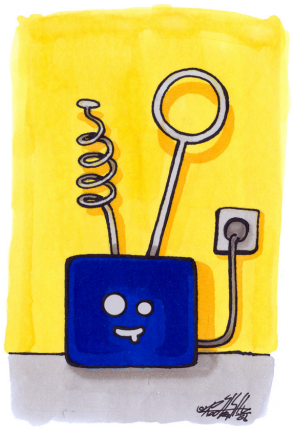
online-go https://online-go.com – excellent and popular website for playing the game online (both real-time and turn-based), solving problems; the site also has introductory material https://online-go.com/learn-to-play-go and exercises for beginners with explanations https://online-go.com/puzzle/2625.

Playing against AI on a computer

Sabaki https://sabaki.yichuanshen.de/ – cross-platform Go board and SGF editor, it can use different AI engines. For example, here are some popular and easy to use engines in increasing order by strength.

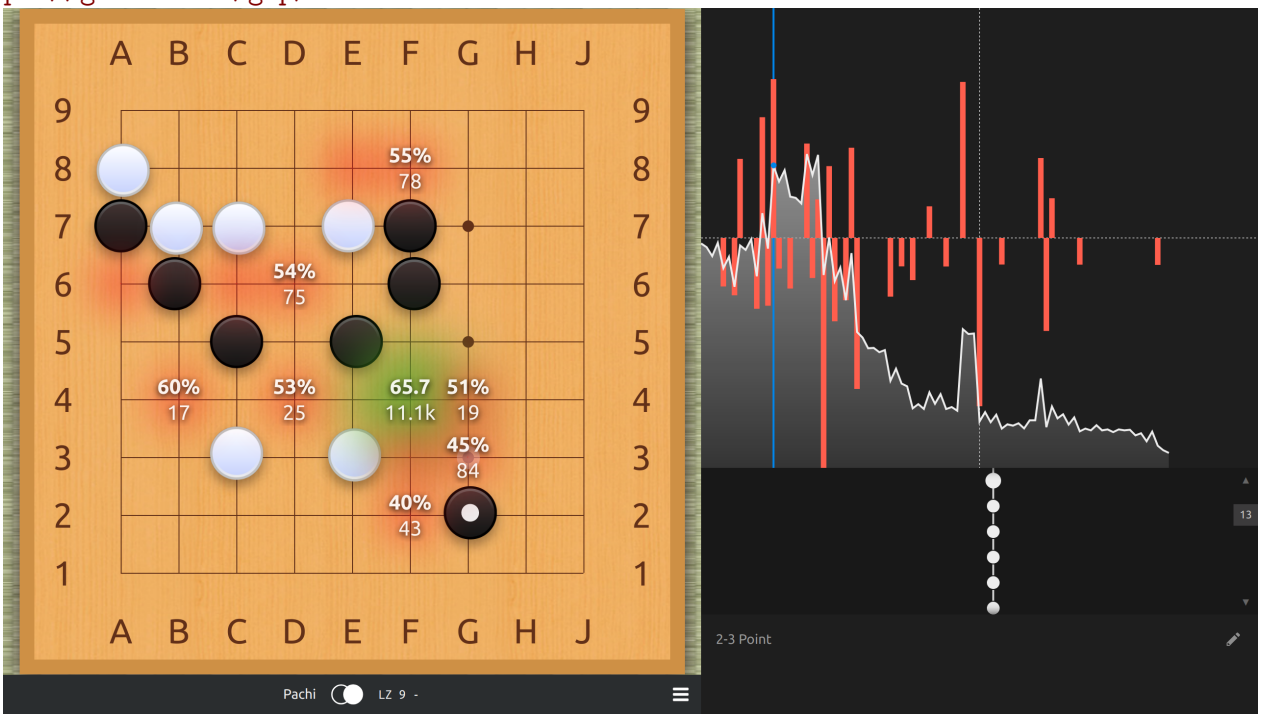


GnuGo A classic Go engine that is used for benchmarking other bots. An example of the good old-fashioned AI, where the expert knowledge about the game is exlicitly written into the software implementation. Not so strong, but very fast. It is not developed any more. https://www.gnu.org/software/gnugo/



Pachi Based on the Monte-Carlo tree search, Pachi is a reasonably strong Go engine. Its strength can be increased by giving more time for the random playouts. the playout policy is not totally random. It is improved by 3×3 patterns and other tactical checks. It is still maintained and it now has a deep convolutional neural network component. http://pachi.or.cz/

Leela combines Monte-Carlo tree search with deep learning techniques. https://www.sjeng.org/leela.html **Leela Zero** is a faithful reimplementation of AlphaGo Zero, so it requires no human knowledge for training. It went superhuman in early 2019. It is widely used for game analysis. https://github.com/gcp/leela-zero



Screenshot of a 9×9 game analysis in progress using the Leela Zero AI engine in Sabaki.

Further Information

Sensei's Library https://senseis.xmp.net – a collaborative web site for all Go related information.

Extracurricular activity

Playing on full board (19×19) in an optional evening class.

Visiting the **Kira Igo Salon** in Goshono area, for playing with local fans of the game. https://kira-igo-salon.amebaownd.com/

Further Reading

Go Introduction

Cho Chikun: **Go – a complete introduction to the game**, 1997. Written by one of the most successful professional players, this short book is a very efficient introduction to the basic techniques. However, information on the professional Go world is dated.

Peter Shotwell: **Go! More than a game**, 2003. One of the many excellent beginner Go books, distinguished by describing the historical, art and scientific context as well.

Go Masters

Hajin Lee: **Outside the Board: Diary of a Professional Go Player**, 2016. Thoughts of a young professional player, both about the game and life in general, in her process of searching for a second career.

Cho Hun-hyun: **Go with the Flow: How the Great Master of Go Trained His Mind**, 2018. Autobiography, with great insights about the thinking process of a professional player.

Go in Literature

Yumi Hotta, Andy Nakatani, Takeshi Obata: **Hikaru No Go Vol 1-24**, 1999-2003. Manga and anime that made many kids start playing Go. Also, many people outside Asia were inspired by the series.

Shan Sa: **The Girl Who Played Go** 2003. It tells a story of a young Chinese girl talented in Go playing with a Japanese officer during the Japanese occupation of Manchuria.

Sung-Hwa Hong: **First kyu**, 1999. A coming of age story of a Go player in Korea.

Trevanian: **Shibumi** 1979. Classic spy novel. The game of Go appears explicitly several times, and it is also a deep underlying metaphor for the whole story.

Yasunari Kawabata: **The Master of Go**, 1954. A semi-fictional chronicle of a half a year long professional game from 1938.

Hermann Hesse: **The Glass Bead Game**, 1943. Inspired by Go (though the game in the book is different and its rules are not defined precisely), the book describes a society that values scholarly activity highly. It is a fictional biography of a player.

Psychology

Daniel Kahneman: **Thinking, Fast and Slow** 2011. This great analysis of intuitive and calculational thinking has a much greater scope, but it does apply to board games as well.

Joshua Waitzkin: **The Art of Learning: An Inner Journey to Optimal Performance** 2007. Former chess prodigy describes his chess studies and tournaments. Then, he shows how the chess expertise could be transferred to martial arts.

Fernand Gobet, Jean Retschitzki, Alex de Voogt: **Moves in mind** 2004. An excellent review of psychological studies of abstract board games (with main focus on chess).

Fred Waitzkin: **Searching for Bobby Fischer: The Father of a Prodigy Observes the World of Chess** 1988. Aforementioned Josh Waitzkin's father tells the same story from the parent's perspective.

Mathematics, Computing, Artificial Intelligence

Peter Norvig, Stuart Russell: **Artificial Intelligence: A Modern Approach, 3rd Edition** 2009. De facto standard textbook on AI.

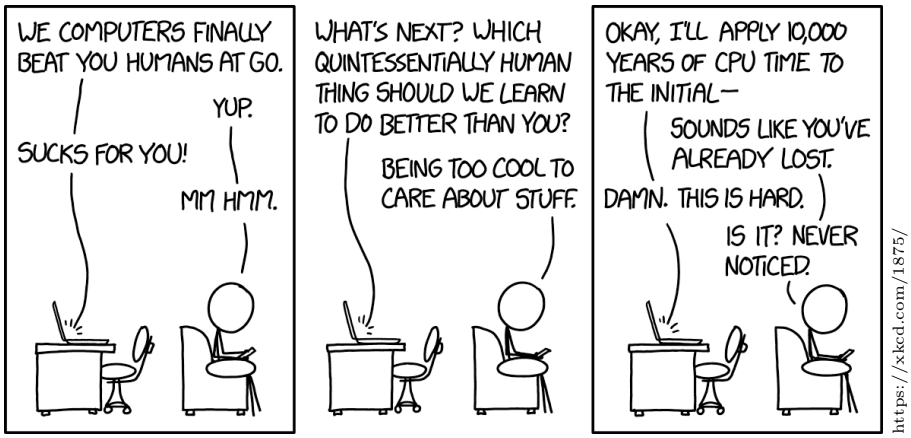
Max Tegmark: **Life 3.0: Being Human in the Age of Artificial Intelligence** 2017. Exploring the implications of the AI revolution with constructive optimism.

Jorge Luis Borges: **The Library of Babel**, 1941. This famous short story describes the nightmarish effects of combinatorial explosions. In a library containing all possible 410 pages long books it is practically impossible to find a meaningful book. This is analogous to collection of all possible games. Only a vanishingly small fraction of the collection contain games that are interesting for us, that arise from a duel between two human players.

Documentary

The Surrounding Game https://www.surroundinggamemovie.com/ 2018. The first feature film about the game itself in the context of setting up a professional system in America. (Shooting of the movie antedates AlphaGo.)

AlphaGo Movie https://www.alphagomovie.com/ 2017. The chronicles of AlphaGo's development and success. It is not a technical documentary, but a deeply moving story about humans striving for excellence.


WE COMPUTERS FINALLY BEAT YOU HUMANS AT GO.
SUCKS FOR YOU!
YUP.
MM HMM.
WHAT'S NEXT? WHICH QUINTESSENTIALLY HUMAN THING SHOULD WE LEARN TO DO BETTER THAN YOU?
BEING TOO COOL TO CARE ABOUT STUFF.
OKAY, I'LL APPLY 10,000 YEARS OF CPU TIME TO THE INITIAL—
DAMN. THIS IS HARD.
SOUNDS LIKE YOU'VE ALREADY LOST.
IS IT? NEVER NOTICED.
https://xkcd.com/1875/